Lab W1D6 Solutions

Q1.Solution

A.

Algorithm sort(A,n)

Input Array A with n length

Output Array A with sorted one color to one side and the other to the other side

If n=0 or n=1 return A

B , R 🡨 empty sequences

For i🡨0 to n-1 do

If A[i]= Blue then

a🡨A.removeFirst()

B.insert(a)

Else

b🡨A.removeFirst()

R.insert(b)

Return BUR

Yes it is in place. And time complexity is O(n)

B.

Algorithm sort(A,n)

Input Array A with n length

Output Array A with sorted one color to one side and the other to the other side

If n=0 or n=1 return A

B ,G, R 🡨 new empty sequences

For i🡨0 to n-1 do

If A[i]= Blue then

a🡨A.removeFirst()

B.insert(a)

Else if A[i]= Green

b🡨A.removeFirst()

G.insert(b)

Else

c🡨A.removeFirst()

R.insert(c)

Return BUGUR

Yes it is in place and the time complexity is O(n).

c.

Algorithm sort(A,n)

Input Array A with n length

Output Array A with sorted one color to one side and the other to the other side

If n=0 or n=1 return A

B ,G, R,Y 🡨 new empty sequences

For i🡨0 to n-1 do

If A[i]= Blue then

a🡨A.removeFirst()

B.insert(a)

Else if A[i]= Green

b🡨removeFirst()

G.insert(b)

Else if A[i]=Red

c🡨A.removeFirst()

R.insert(c)

Else

d🡨A.removeFirst()

Y.insert(d)

Return BUGURUY

Yes it is in Place and the time complexity of this program is O(n)

Q2. Solution

A.

Array A={1, 2, 3, 4, 5, 6, 7, ,8, 9};

Pivot is the median ;

To find A median:

Position of median is floor[ (0 + 9)/2] = 4;

A[0]= 1 , A[4]=5 and A[9]=9

So, Median of 1,5,9 is 5

We take 5 as pivot

-first we swap pivot with last element

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 1 | 2 | 3 | 4 | 9 | 6 | 7 | 8 | 5 |

I j

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 1 | 2 | 3 | 4 | 9 | 6 | 7 | 8 | 5 |
|  |  |  | j | i |  |  |  |  |

Now

Swap (I , pivot)

Now the Array is sorted as

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

Good self call because L=4 and G= 4 so,(3/4)\*4= 3 which is less than ¾ of the Array size.

B.

A={8, 7, 6, 5, 4, 3, 2, 1, 9}

Median position is floor[(0+9)/2] =4

A[0]=8 , A[9]= 9; A[4]=4;

So, median of 8,9,4 is 4;

So, our pivot is 4

Swap pivot and last element

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 8 | 7 | 6 | 5 | 9 | 3 | 2 | 1 | 4 |

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 8 | 7 | 6 | 5 | 9 | 3 | 2 | 1 | 4 |
| i |  |  |  |  |  |  | j |  |

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 1 | 7 | 6 | 5 | 9 | 3 | 2 | 8 | 4 |
|  | i |  |  |  |  | j |  |  |

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 1 | 2 | 6 | 5 | 9 | 3 | 7 | 8 | 4 |
|  |  | i |  |  | j |  |  |  |
| 1 | 2 | 3 | 5 | 9 | 6 | 7 | 8 | 4 |
|  |  |  | j | i |  |  |  |  |

I and j are crossed over

Swap(I,pivot)

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 1 | 2 | 3 | 9 | 5 | 6 | 7 | 8 | 4 |
|  |  |  |  |  |  |  |  |  |

The sorted Array is

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

C.

A={9, 1, 8, 2, 7, 3, 6, 4, 5}

The position is floor[(0+9)/2]=4

A[0]=9 , A[9]=5, A[4]=7

So, median of 9,5,7 is 5, which is our pivot

Swap(p, last element)

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 9 | 1 | 8 | 2 | 7 | 3 | 6 | 4 | 5 |
| i |  |  |  |  |  |  | j | p |

Swap(9,4)

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 4 | 1 | 8 | 2 | 7 | 3 | 6 | 9 | 5 |
|  |  | i |  |  | j |  |  | p |

Swap(8,3)

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 4 | 1 | 3 | 2 | 7 | 8 | 6 | 9 | 5 |
|  |  |  | j | i |  |  |  | p |

Swap(I,p)

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 4 | 1 | 3 | 2 | 5 | 8 | 6 | 9 | 7 |
|  |  |  |  |  |  |  |  |  |

Now numbers less than pivot are on L={4,1,3,2} E={5} G={8,6,9,7}

And then Lets do it recursively for L={4, 1, 3, 2}

Position floor[(0+4)/2]=2

A[0]=4 , A[4]=2 A[2]=3

Median of 4,2,3 is 3 our pivot is 3.

Swap(p,last element)

|  |  |  |  |
| --- | --- | --- | --- |
| 4 | 1 | 2 | 3 |
| i |  | j | p |

|  |  |  |  |
| --- | --- | --- | --- |
| 2 | 1 | 4 | 3 |
|  | j | i | p |

Swap(I,p)

|  |  |  |  |
| --- | --- | --- | --- |
| 2 | 1 | 3 | 4 |

Now we got L={2,1} E={3} G={4}

Pick p=1;

|  |  |
| --- | --- |
| 2 | 1 |
|  | p |

The G={2} one element is sorted by itself.

So, L=EUG={1,2}, LUE U G={1,2,3,4},

This is L={1,2,3,4} , E={5},

Now Lets do it for G={8,6,9,7};

Position floor[(0+4)/2]= 2

A[0]=8, A[4]= 7 , A[2]=9

And median of 8,7,9 is 7

Swap(p, last Element)

|  |  |  |  |
| --- | --- | --- | --- |
| 8 | 6 | 9 | 7 |
| i |  | j | p |

|  |  |  |  |
| --- | --- | --- | --- |
| 8 | 6 | 9 | 7 |
| i | j |  | p |

Swap(8,6)

|  |  |  |  |
| --- | --- | --- | --- |
| 6 | 8 | 9 | 7 |
| j | i |  | p |

I and j crossed over so swap(I,p)

|  |  |  |  |
| --- | --- | --- | --- |
| 6 | 7 | 9 | 8 |
|  |  |  |  |

Now we have L={6} E={7} G={9,8}

G={9, 8}

Pick 9 as pivot

We left with L={8} and one element array is sorted by itself.

So, G=LUE={8, 9} and G=LUEUG={6,7,8,9}

So far we have L={1,2,3,4} , E={5}, And Now G={6,7,8,9}

Overall process is

LUEUG={1,2,3,4,5,6,7,8,9}

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

D.

A={5, 1, 4, 2, 3 ,9, 7, 6 ,8}

The position is floor[(0+9)/2]=4

A[0]=5 , A[9]=8, A[4]=3

So, median of 5,8,3 is 5, which is our pivot is 5

Swap (p, last element)

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 8 | 1 | 4 | 2 | 3 | 9 | 7 | 6 | 5 |
| i |  |  |  | j |  |  |  | p |

Swap(8,3)

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 3 | 1 | 4 | 2 | 8 | 9 | 7 | 6 | 5 |
|  |  |  | j | i |  |  |  | p |

I and j are crossed over then now swap(p,i)

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 3 | 1 | 4 | 2 | 5 | 9 | 7 | 6 | 8 |
|  |  |  |  |  |  |  |  |  |

Now we have L={3,1,4,2} E={5} G={ 9,7,6,8}

Lets do it recursively

For L and G separately

L={3,1,4,2};

Position Floor[(0+4)/2]= 2

A[0]=3 A[4]=2 A[2]=4 and median of 3,2,4 is 3 which is our pivot

Swap(p, last element)

|  |  |  |  |
| --- | --- | --- | --- |
| 2 | 1 | 4 | 3 |
| i | j |  | P |

Swap(2,1)

|  |  |  |  |
| --- | --- | --- | --- |
| 1 | 2 | 4 | 3 |
| j |  | i | p |

I and j is crossed over so,

Swap( I,p)

|  |  |  |  |
| --- | --- | --- | --- |
| 1 | 2 | 3 | 4 |
|  |  |  |  |

Now L={1,2} E={3} G={4}

L is sorted by default so L=LUEUG={1,2,3,4}

And Now lets do for G={ 9,7,6,8}

Position Floor[(0+4)/2]= 2

A[0]=9 A[4]=8 A[2]= 6

So median of 9,8,6 is 8 which is our pivot

So lets swap(p, last element)

|  |  |  |  |
| --- | --- | --- | --- |
| 9 | 7 | 6 | 8 |
| i |  | j | p |

Swap(9,6)

|  |  |  |  |
| --- | --- | --- | --- |
| 6 | 7 | 9 | 8 |
|  | j | i | p |

I and j crossed over so swap(I,p)

|  |  |  |  |
| --- | --- | --- | --- |
| 6 | 7 | 8 | 9 |
|  |  |  |  |

Now G is now sorted L={6,7} E={8} G={9}

Since L is sorted by default

G=LUEUG={ 6, 7, 8, 9}

So far we have L={1,2,3,4} E={5} G={ 6, 7, 8, 9}

So, LUEUG={ 1, 2, 3, 4, 5, 6, 7, 8, 9}

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

Q3.Solution

A.

Array A={1, 2, 3, 4, 5, 6, 7, ,8, 9}

Pick pivot randomly and I pick 5

-first we swap pivot with last element

K=5

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

I

Now j

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 1 | 2 | 3 | 4 | 9 | 6 | 7 | 8 | 5 |

Now since K=5

L={1,2,3,4} G={6,7,8,9}

Now k<=|L|+|E|=4+1=5, so, 5<=5

So, 5 is the Answer;

B.

A={8, 7, 6, 5, 4, 3, 2, 1, 9}

Pick pivot 4

K=3 s={8, 7, 6, 5, 4, 3, 2, 1, 9}

K=3 s={3,2,1}

K = 3 s={1,2,3}

So, the third element is 3;

C.

K=8 s={9, 1, 8, 2, 7, 3, 6, 4, 5 }

P=5

K= 3 s={ 9, 8, 7, 6}

K=3 S={6, 7, 8 }

So, the eighth element is 8,

D.

K=5 s= {5, 1, 4, 2, 3, 9, 7, 6, 8}

P=5

K=5 since |L| < k<= |L|+|E| => 5<=4+1 => 5<=5

So the fifth element is 5.

Q4.Solution

A.

Lets say

Good Self - call : the size of L and G are less than 2n/3.

Bad Self - call : the size of L and G has size greater than or equal to 2n/3.

7 2 9 4 3 7 6 1

L={ 2, 4 , 3 , 1}

G= { 7, 9, 7 }

2(8)/3= 5 ,so size of L and G must be less than 5 so , the size of L= 4 and size of G= 3 so, it is Good Self- Call;

B.

7 2 9 4 3 7 6 1

L={1}

G={ 7 , 9, 4 , 3, 7, 6 }

So, size of L and G must less than 5 .

So, size of G is 6 . It is a bad self-call

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16

Green is Bad pivot.

Red is good pivot.

Order is O(n log n).